
MINEX III Report Card
Template Generator IDWorld+0005



Last Updated: April 17, 2018

Participant Details

Company: Beijing IDWorld Science and Technology Development Co., Ltd
Provided CBEFF PID: 0069 0005

Date Application Received: 07/28/2017

Date First Submitted: 09/13/2017 (as generator version 0001)

Date Validated: 09/19/2017

Date Completed: 09/19/2017

Library	Size (bytes)	MD5 Checksum
libminexiii_IDWorld_0005.so	2867040	d9d5c79e9c0a927d22ebaa50a418ee8d

Compliance Test Results

The following presents **PIV compliance** results per the criteria detailed in [NIST Special Publication 800-76-2: Biometric Specifications for Personal Identity Verification](#).

It also includes **MINEX III compliance** results per the criteria detailed in sections 4 through 8 of the [Minutia Interoperability Exchange \(MINEX\) III Test Plan and Application Programming Interface](#).

PIV: FAIL

- All certified matchers must be able to match templates from this template generator with an $\text{FNMR}_{\text{FMR}}(0.01) \leq 0.01$ using two fingers (4.5.2.2-3). ✓
- Average template creation time must be no more than 500 milliseconds (4.5.2.2-2). ✓
- Minutia density plots derived from generated templates do not exhibit a periodic, grid-like, or geometric structure. ✗ (See Section 3.5)
- If matcher also submitted, matcher is PIV Level Two compliant. ✗

MINEX III: FAIL

- Must pass MINEX III validation. ✓
- Must be PIV compliant. ✗
- No more than two compliant template generators from the submitting organization, or its subsidiaries, acquisitions, or mergers allowed (8.8). ✓
- If matcher also submitted, matcher is MINEX III compliant. ✗

Notes

- This report will be updated as new matching algorithms and template generators pass the compliance test. These updates will not change the PASS/FAIL decision above.
- NIST reserves the right to decertify a template generator if it later discovers the template generator violates MINEX III or PIV specifications in some previously undetected way.
- This submission is not compliant, and is therefore *not* a member of the pooled DET curves published throughout all MINEX III report cards.

Contents

Participant Details	1
Compliance Test Results	1
Notes	1
1 Introduction	4
2 Methodology	4
2.1 Dataset	4
2.2 Accuracy Metrics	4
2.3 Uncertainty Estimation	4
2.4 Interoperability	5
3 Results	6
3.1 Single Finger	6
3.2 Two Finger	9
3.3 Template Creation Times	11
3.4 Minutia Counts	12
3.5 Minutia Density Plots	14
3.6 Comparison to Ongoing MINEX	15
4 Performance Tables	16
5 References	20

List of Figures

1 MINEX III Interoperability Test Setup	5
2 DET (Single Finger)	6
3 DET (Right Index)	7
4 DET (Left Index)	7
5 FNMR @ FMR = 0.01 (Single Finger)	8
6 DET (Two Finger)	9
7 FNMR @ FMR = 0.01 (Two Finger)	10
8 Template Creation Times	11
9 Minutia Count Probability	12
10 Minutia Count Cumulative Summation	13
11 2D Minutia Placement Density Functions	14

List of Tables

1 Single finger	15
2 Two finger	15
3 Single finger	16
4 Right index finger	17
5 Left index finger	18

6 Two finger 19

1 Introduction

Testing is performed at a NIST facility. Each participant’s submission is validated by NIST before undergoing full testing to ensure it operates correctly. If the matcher passes the validation procedure, it is then used to compare standard fingerprint templates. Performance is assessed against templates created by a template generator submitted by the participant as well as templates created by other MINEX III compliant template generators.

2 Methodology

Testing is performed at a NIST facility. Each participant’s submission is validated by NIST before undergoing full testing to ensure it operates correctly. If the template generator passes the validation procedure, performance is assessed by using MINEX III compliant matching algorithms to compare templates created by the template generator. These matchers were submitted to the ongoing MINEX III program by various participants.

2.1 Dataset

Testing is performed over a single dataset of sequestered fingerprint images. The images were collected by U.S. Visit at ports of entry into the United States. They consist of Live-scan plain impressions of left and right index fingers. WSQ [1] compression was applied to all images at a ratio of 15:1. The most recent capture of each subject was treated as the authentication sample, and the next most recent as the enrolled sample.

The dataset was divided into 533 767 mated and 1 067 530 non-mated subject pairings. Since both left and right index fingerprints are available for each subject, this provides 1 061 657 mated and 2 127 712 non-mated single-finger comparisons (after database consolidation). When left and right index fingers are fused at the score level [3, 8], the sets condense to 530 394 mated and 1 062 814 non-mated comparison scores.

2.2 Accuracy Metrics

Core matching accuracy is presented in the form of Detection Error Tradeoff (DET) plots [7], which show the trade-off between the False Match Rate (FMR) and the False Non-Match Rate (FNMR) as a decision threshold is adjusted. Formally, let m_i ($i = 1 \dots M$) be the i th mated comparison score, and n_j ($j = 1 \dots N$) the j th non-mated comparison score. Then the statistics are

$$\text{FMR}(\tau) = \frac{1}{N} \sum_{j=1}^N \mathbb{1}\{n_j \geq \tau\}, \quad (1)$$

$$\text{FNMR}(\tau) = \frac{1}{M} \sum_{i=1}^M \mathbb{1}\{m_i < \tau\}. \quad (2)$$

where $\mathbb{1}\{A\}$ is the indicator [4] of event A . Equations 1 and 2 define the curve parametrically with the decision threshold, τ , as the free parameter. In some figures and tables, FNMR is presented as a function of FMR. This relationship is determined by

$$\text{FNMR}_{\text{FMR}}(\alpha) = \min_{\tau} \{ \text{FNMR}(\tau) \mid \text{FMR}(\tau) \leq \alpha \}, \quad (3)$$

which reads as the smallest FNMR that can be achieved while maintaining an FMR less than or equal to α , the targeted FMR. This method of relating the two error statistics ensures FNMR is well-defined for all $0 \leq \alpha \leq 1$. It also imposes a natural penalty on matching algorithms that produce heavily discretized scores.

2.3 Uncertainty Estimation

Some figures in this report include boxplots that convey the uncertainty associated with a statistic. The boxplots are intended to show the expected variation in the observed value if one assumes repeated iid sampling from the same population. They are not intended to reflect how the statistic might change over different test data or even different sampling strategies over the same data.

Estimates of uncertainty are computed using the Wilson Score method [10] which overcomes certain problems associated with applying the Central Limit Theorem to a discretized estimator. We make several simplifying assumptions when applying the method to biometric identification. Most notably, separate searches against the same enrollment database are treated as independent samples, yet we know positive correlations exist due to Doddingtons Zoo [2]. We also report estimates of the variability of FNIR at a fixed FPIR when in fact it is the decision threshold that is fixed. Uncertainty with respect to what decision threshold corresponds to the targeted FPIR results in increased uncertainty about the true value of FNIR. However, our estimates of FPIR are fairly



Figure 1: MINEX III Interoperability Test Setup

tight due to the large number of non-mated searches performed, so they are not expected to have a large impact on the estimates.

2.4 Interoperability

Interoperability is tested in a manner similar to *Scenario 1* from the [MINEX Evaluation Report \[5\]](#) (see Figure 1). An enrolment template is prepared using submission X. Submission Y is used to prepare the authentication template and perform the match. The authentication template is always prepared by the same submission used to compare the templates. However, enrolment templates need not originate from the same submission. When they do, we refer to as “native” mode.

3 Results

This section details the performance of template generator IDWorld+0005. Sections 3.1 and 3.2 present accuracy results for single finger and two finger matching respectively. Section 3.4 presents information on the number of minutia the template generator finds in the samples.

3.1 Single Finger

Single finger comparison results show the combined results for left and right index comparisons. For reference, *NIST Special Publication 800-76-2* requires that the template generator achieve an accuracy of $\text{FNMR}_{\text{FMR}(0.01)} \leq 0.01$ against all compliant matchers.

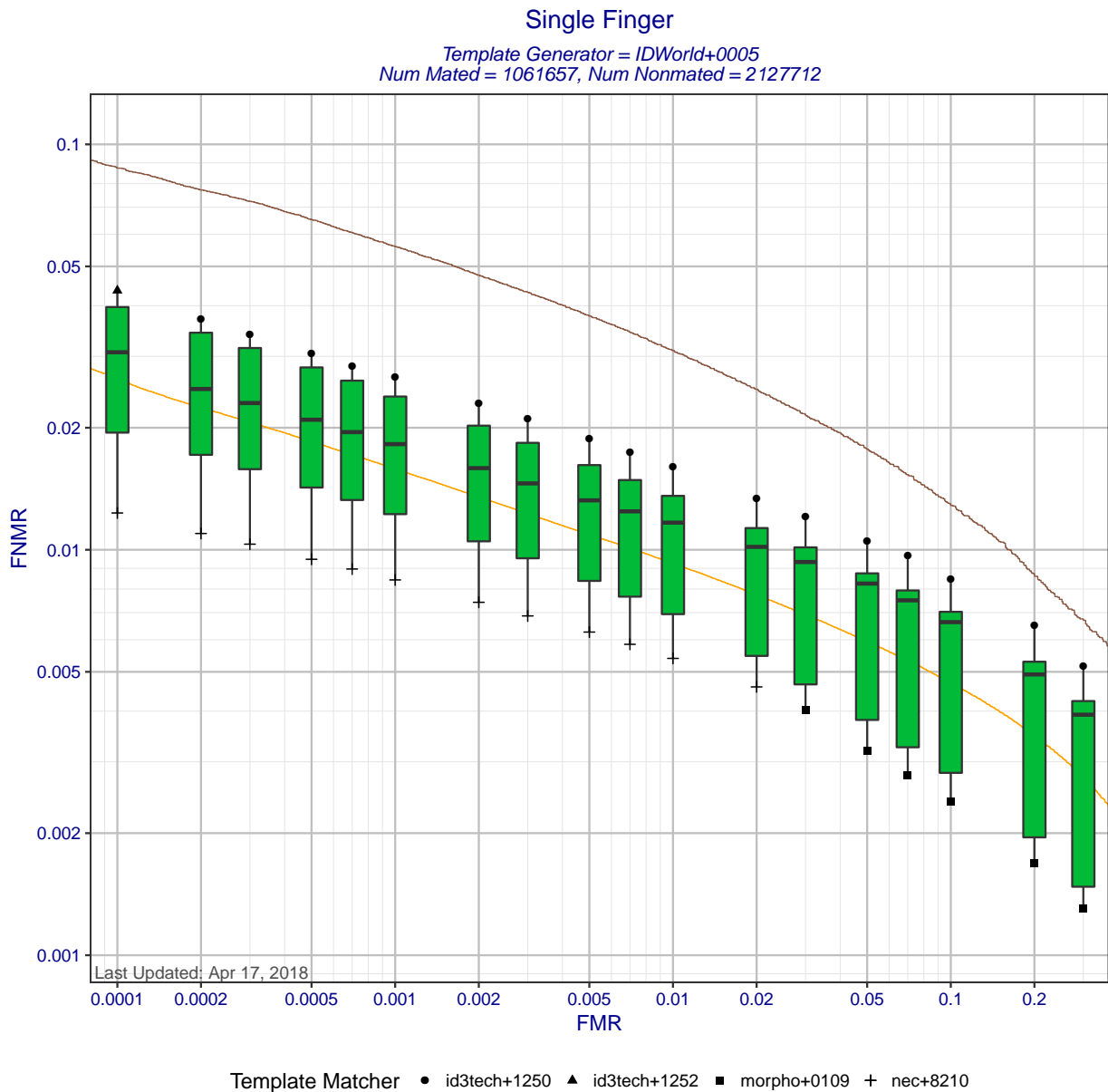


Figure 2: Single finger DET statistics for template generator IDWorld+0005. Each box shows the distribution of FNMRs at a fixed FMR across different matchers. The whisker ends show the minimum and maximum FNMRs. The brown curve shows the DET curve when the matcher and template generators were submitted by the same participant. The orange DET curve shows pooled performance when all matchers compare templates created by IDWorld+0005.

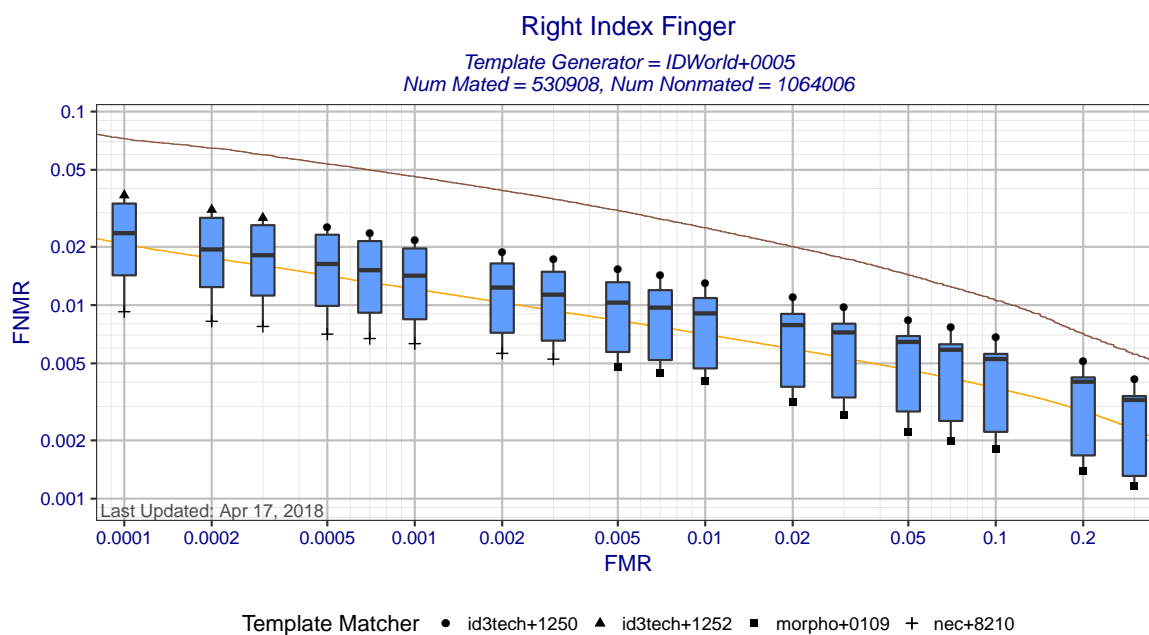


Figure 3: Right Index Finger DET statistics for template generator IDWorld+0005. Each box shows the distribution of FNMRs at a fixed FMR across different matchers. The whisker ends show the minimum and maximum FNMRs. The brown curve shows the DET curve when the matcher and template generators were submitted by the same participant. The orange DET curve shows pooled performance when all matchers use templates created by IDWorld+0005.

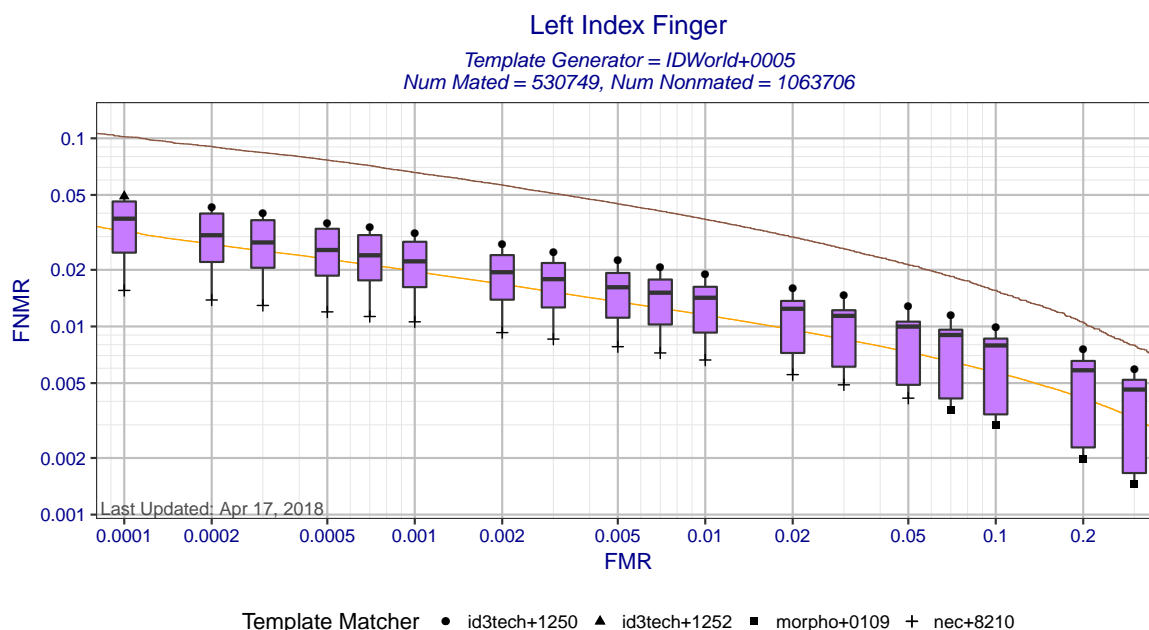


Figure 4: Left Index Finger DET statistics for template generator IDWorld+0005. Each box shows the distribution of FNMRs at a fixed FMR across different template generators. The brown curve shows the DET curve when the matcher and template generators were submitted by the same participant. The orange DET curve shows pooled performance when all matchers use templates created by IDWorld+0005.



Figure 5: Single Finger FNMRs at FMR=0.0001 when MINEX III compliant matchers compare templates created by template generator IDWorld+0005. Each box represents uncertainty about the true FNMR. The box edges mark the 50% confidence intervals while the whiskers mark the 90% confidence intervals. The numbers on the right show the actual computed FNMRs.

3.2 Two Finger

This section presents accuracy when different MINEX III compliant matchers compare templates created by template generator IDWorld+0005. Two finger fusion is achieved by averaging the scores for left and right index fingers for each person.

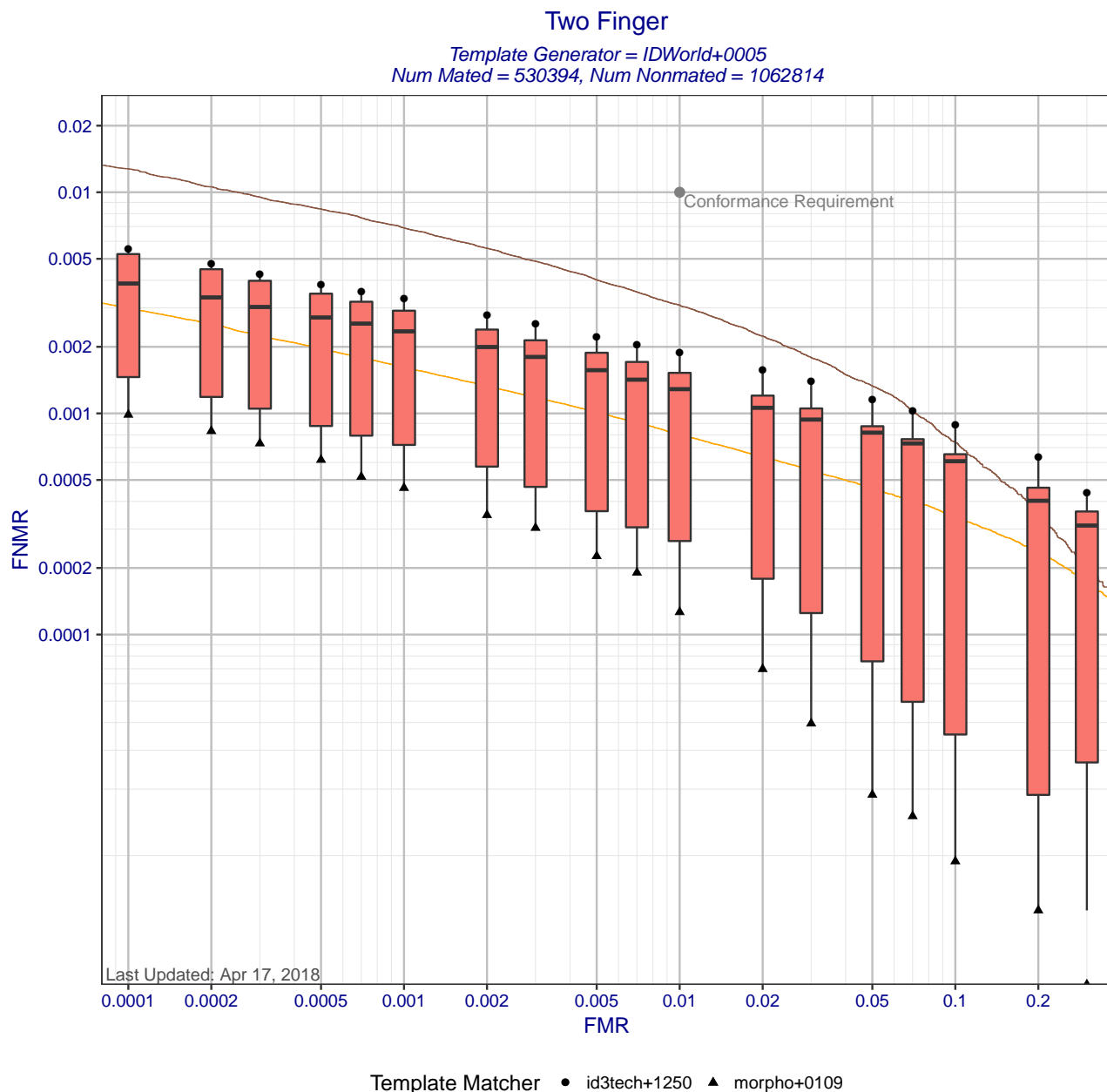


Figure 6: Two Finger DET statistics for template generator IDWorld+0005. Each box shows the distribution of FNMRs at a fixed FMR across different matchers. The whisker ends show the minimum and maximum FNMRs. The brown curve shows the DET curve when the matcher and template generators were submitted by the same participant. The orange DET curve shows pooled performance when all matchers use templates created by IDWorld+0005. Score-level fusion is achieved by averaging the scores for left and right index fingers.

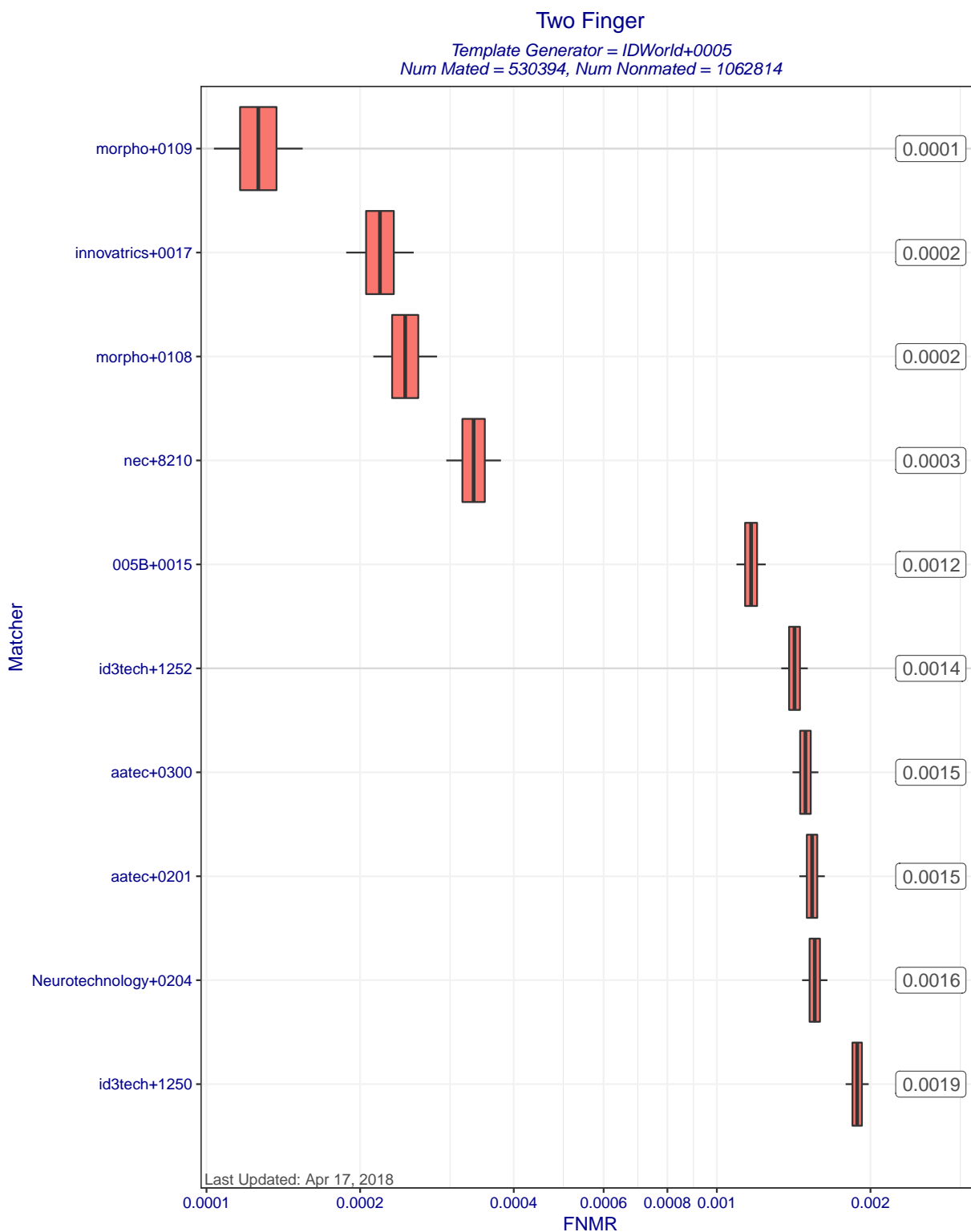


Figure 7: Two Finger FNMR at FMR=0.01 when different matchers compare templates created by template generator IDWorld+0005. Each box represents uncertainty about the true FNMR. The box edges mark the 50% confidence intervals while the whiskers mark the 90% confidence intervals. The numbers on the right show the actual computed FNMRs. Score-level fusion is achieved by averaging the scores for left and right index fingers.

3.3 Template Creation Times

To achieve PIV compliance, the template generator must create templates in no more than 0.5 seconds (500 milliseconds) on average.

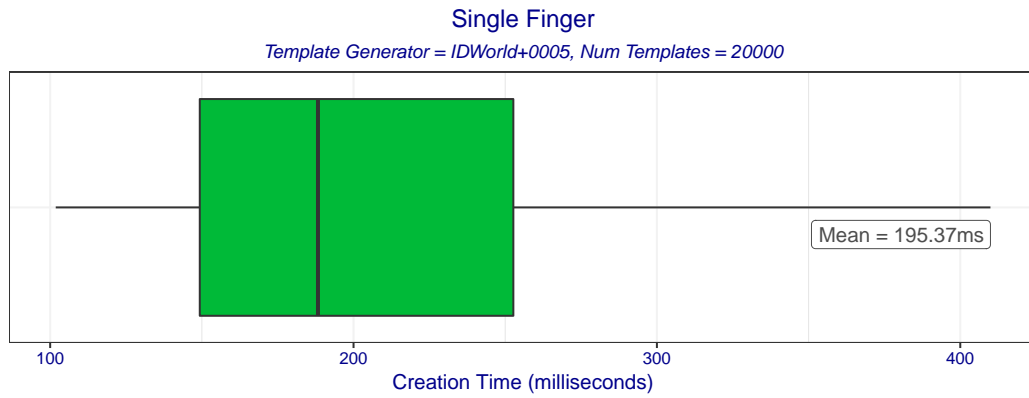


Figure 8: Boxplot of template creation times for template generator IDWorld+0005. The box edges mark the 10th and 90th percentiles while the whiskers mark the maximum and minimum creation times.

3.4 Minutia Counts

This section presents information relating to the number of minutia the template generator finds in fingerprint images. The relative number of minutia found in common fingerprint images has been shown to influence matching outcomes [9, 6].

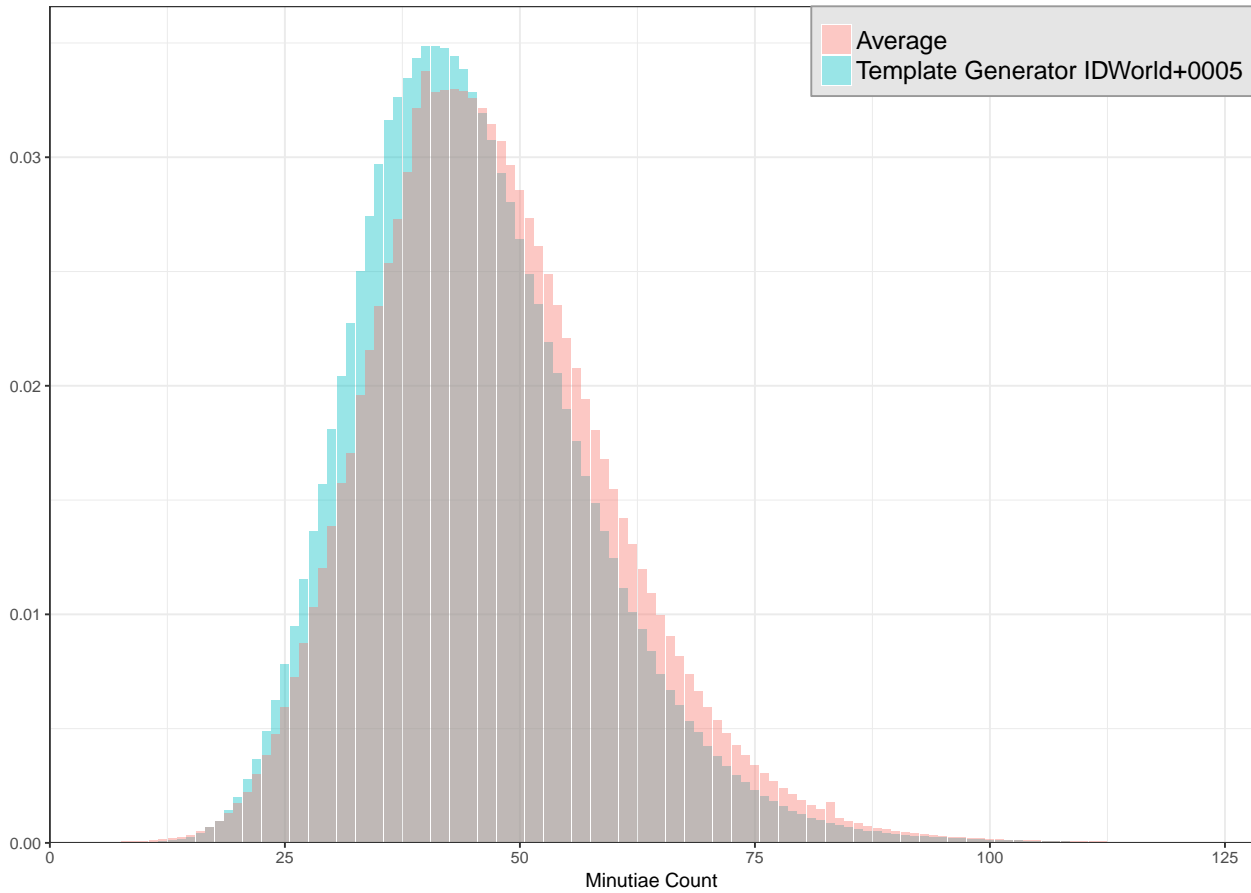


Figure 9: Probability distribution of the number of minutia the template generator found in the samples. The average probability distribution shows the combined distribution of minutia counts across all compliant template generators submitted for MINEX III (i.e., excluding Ongoing MINEX template generators).

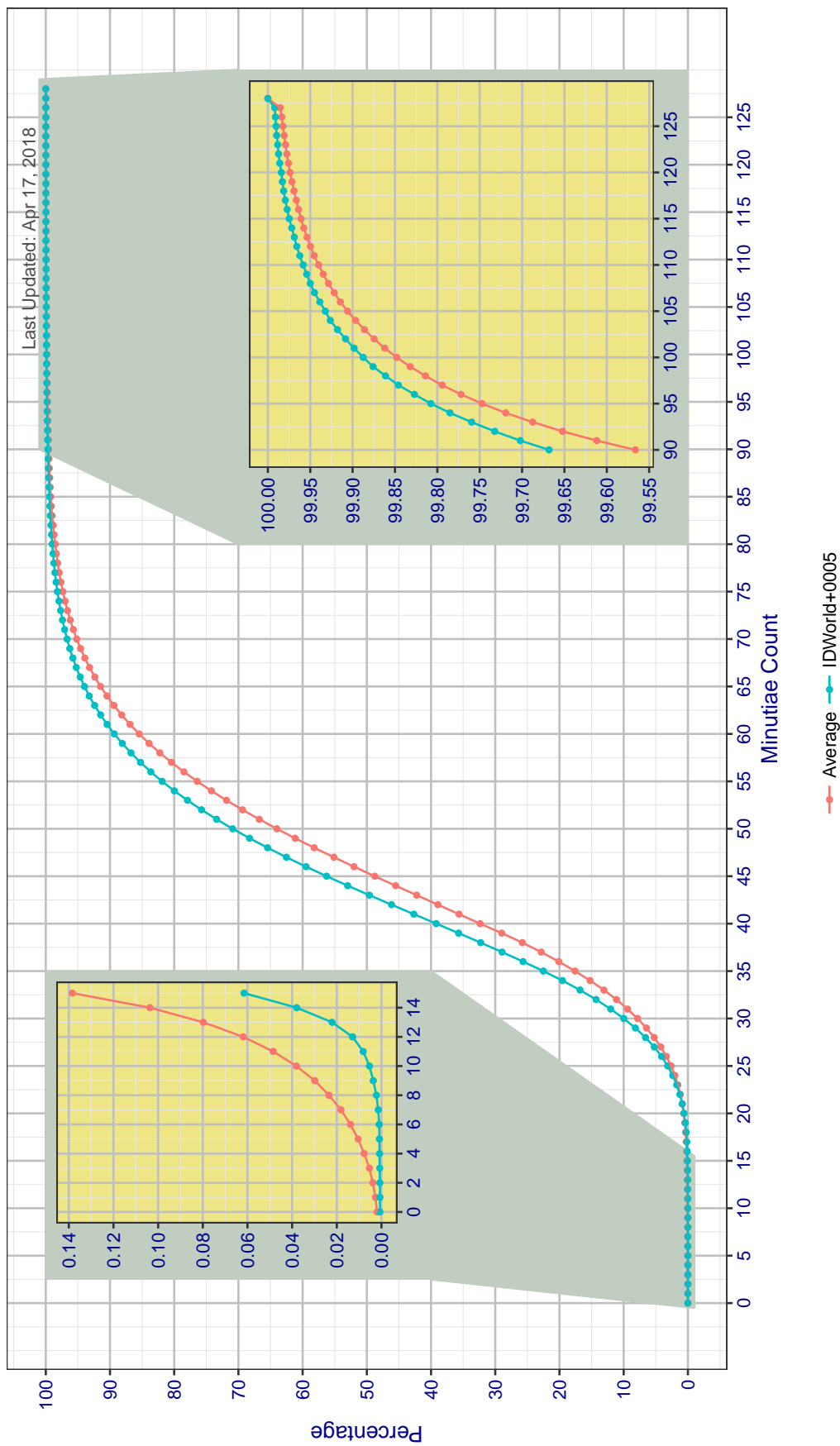


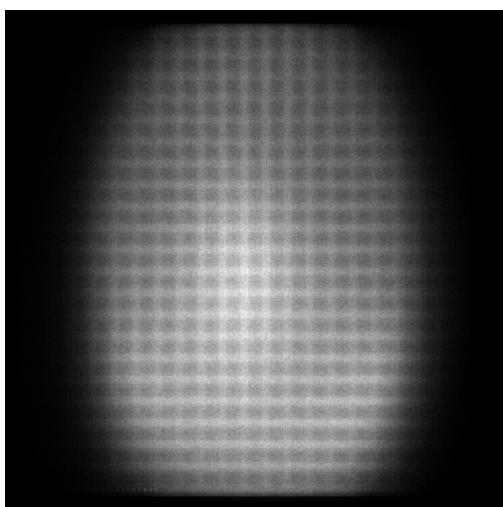
Figure 10: Cumulative summation of the number of minutiae the template generator found in the samples. The average probability distribution shows the combined distribution of minutiae counts across all compliant template generators submitted for MINEX III (i.e., excluding Ongoing MINEX template generators).

3.5 Minutia Density Plots

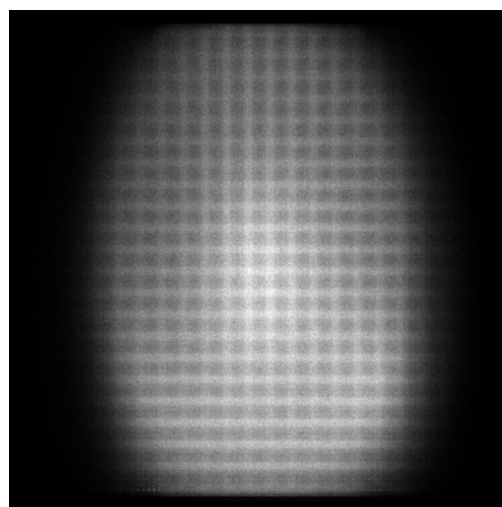
Minutia density plots show where the template generator tends to find minutia in fingerprint images. They are 2D histograms where the degree of illumination at an (x, y) coordinate indicates how frequently the software located a minutiae point at that location. The purpose of showing minutia density plots is to determine whether the template generator exhibits regional preference when locating minutia.

NIST has determined that this template generator produces minutia exhibiting a periodic structure. Periodic structures and other regional preferences are an indication that the template generator is departing from the minutia placement requirements of INCITS 378, clause 5. The expected pattern is a locally uniform distribution, and the appearance of local structure indicates systematic non-conformance with the standard. Given such behavior negatively affects interoperability[9], developers are asked to determine the cause of such behavior – for example, as an artifact of a tiled-based image processing algorithms applied to the input fingerprint image – and to resubmit corrected algorithms.

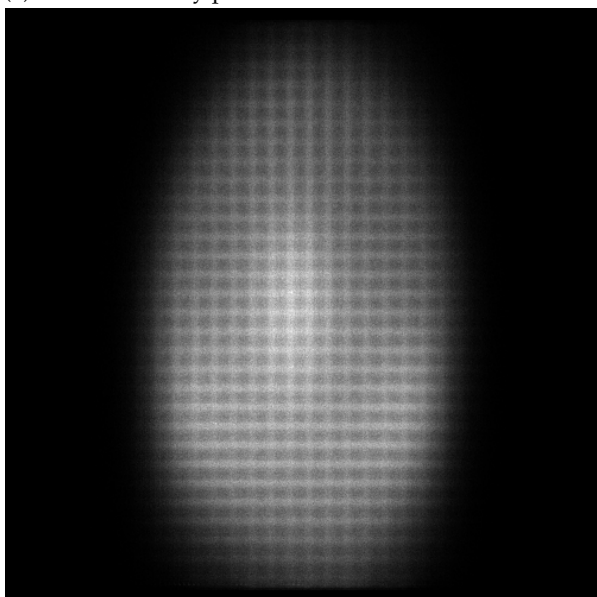
NIST uses a closed-form test to detect high frequency periodic structure by searching for modulation in the 368x368 minutia plot's Fourier representation. The code for this test is available [on GitHub](#).



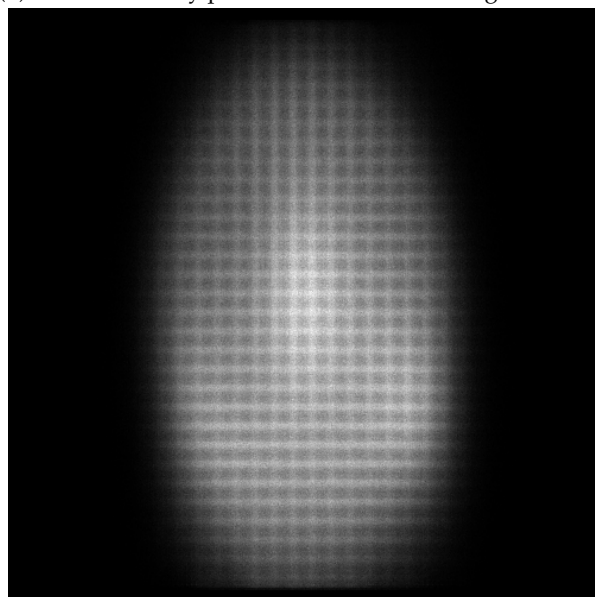
(a) Minutia density plot for 671 619 368x368 left indexes.



(b) Minutia density plot for 671 460 368x368 right indexes.



(c) Minutia density plot for 477 312 500x500 left indexes.



(d) Minutia density plot for 477 317 500x500 right indexes.

Figure 11: 2D Minutia Placement Density Functions.

3.6 Comparison to Ongoing MINEX

MINEX III uses a larger set of comparisons than the older ongoing MINEX evaluation. Although this is generally good because it provides more accurate estimates of performance in MINEX III, it makes it more difficult to directly compare the results in this report to the archived ones from ongoing MINEX. The tables below report DET accuracy at fixed FMRs computed over the same set of comparisons that were used in ongoing MINEX. Ongoing MINEX reported FNMR at $FMR = 0.01$ for two-finger.

Table 1: *Single finger FNMRs at various FMRs when IDWorld+0005 and MINEX III-compliant matchers compare templates created by template generator IDWorld+0005.*

Matcher	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
005B+0015	0.0115 ± 0.0004	0.0183 ± 0.0004	0.0271 ± 0.0005
aatec+0201	0.0121 ± 0.0004	0.0204 ± 0.0005	0.0322 ± 0.0006
aatec+0300	0.0140 ± 0.0004	0.0232 ± 0.0005	0.0371 ± 0.0006
id3tech+1250	0.0166 ± 0.0004	0.0283 ± 0.0005	0.0486 ± 0.0007
id3tech+1252	0.0141 ± 0.0004	0.0265 ± 0.0005	0.0483 ± 0.0007
innovatrics+0017	0.0075 ± 0.0003	0.0161 ± 0.0004	0.0307 ± 0.0006
morpho+0108	0.0096 ± 0.0003	0.0141 ± 0.0004	0.0210 ± 0.0005
morpho+0109	0.0083 ± 0.0003	0.0126 ± 0.0004	0.0188 ± 0.0004
nec+8210	0.0047 ± 0.0002	0.0079 ± 0.0003	0.0125 ± 0.0004
Neurotechnology+0204	0.0144 ± 0.0004	0.0259 ± 0.0005	0.0449 ± 0.0007
IDWorld+0005	0.0343 ± 0.0006	0.0617 ± 0.0008	0.0983 ± 0.0010

Table 2: *Two finger FNMRs at various FMRs when IDWorld+0005 and MINEX III-compliant matchers compare templates created by template generator IDWorld+0005.*

Matcher	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
005B+0015	0.0012 ± 0.0002	0.0021 ± 0.0002	0.0031 ± 0.0003
aatec+0201	0.0015 ± 0.0002	0.0027 ± 0.0002	0.0050 ± 0.0003
aatec+0300	0.0016 ± 0.0002	0.0030 ± 0.0003	0.0053 ± 0.0003
id3tech+1250	0.0019 ± 0.0002	0.0033 ± 0.0003	0.0060 ± 0.0004
id3tech+1252	0.0015 ± 0.0002	0.0028 ± 0.0002	0.0056 ± 0.0003
innovatrics+0017	0.00024 ± 0.00007	0.0010 ± 0.0001	0.0023 ± 0.0002
morpho+0108	0.0005 ± 0.0001	0.0010 ± 0.0001	0.0015 ± 0.0002
morpho+0109	0.00037 ± 0.00009	0.0009 ± 0.0001	0.0016 ± 0.0002
nec+8210	0.00039 ± 0.00009	0.0007 ± 0.0001	0.0011 ± 0.0002
Neurotechnology+0204	0.0015 ± 0.0002	0.0029 ± 0.0003	0.0050 ± 0.0003
IDWorld+0005	0.0036 ± 0.0003	0.0083 ± 0.0004	0.0153 ± 0.0006

4 Performance Tables

The following tables present accuracy numbers, including estimates of uncertainty in the form of 90% confidence bounds. These tables are provided because most of the figures in the main body of the report do not present actual accuracy numbers.

Table 3: *Single finger FNMRs at various FMRs when IDWorld+0005 and MINEX III-compliant matchers compare templates created by template generator IDWorld+0005.*

Matcher	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
005B+0015	0.0115 ± 0.0002	0.0168 ± 0.0002	0.0253 ± 0.0003
aatec+0201	0.0119 ± 0.0002	0.0197 ± 0.0002	0.0312 ± 0.0003
aatec+0300	0.0131 ± 0.0002	0.0213 ± 0.0002	0.0333 ± 0.0003
id3tech+1250	0.0160 ± 0.0002	0.0266 ± 0.0003	0.0425 ± 0.0003
id3tech+1252	0.0137 ± 0.0002	0.0248 ± 0.0002	0.0436 ± 0.0003
innovatrics+0017	0.0071 ± 0.0001	0.0148 ± 0.0002	0.0302 ± 0.0003
morpho+0108	0.0069 ± 0.0001	0.0115 ± 0.0002	0.0178 ± 0.0002
morpho+0109	0.0060 ± 0.0001	0.0102 ± 0.0002	0.0163 ± 0.0002
nec+8210	0.0054 ± 0.0001	0.0084 ± 0.0001	0.0123 ± 0.0002
Neurotechnology+0204	0.0143 ± 0.0002	0.0253 ± 0.0003	0.0421 ± 0.0003
IDWorld+0005	0.0310 ± 0.0003	0.0559 ± 0.0004	0.0875 ± 0.0005

Table 4: *Right index finger FNMRs at various FMRs when IDWorld+0005 and MINEX III-compliant matchers compare templates created by template generator IDWorld+0005.*

Matcher	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
005B+0015	0.0087 ± 0.0002	0.0126 ± 0.0003	0.0189 ± 0.0003
aatec+0201	0.0094 ± 0.0002	0.0160 ± 0.0003	0.0259 ± 0.0004
aatec+0300	0.0103 ± 0.0002	0.0173 ± 0.0003	0.0277 ± 0.0004
id3tech+1250	0.0131 ± 0.0003	0.0218 ± 0.0003	0.0363 ± 0.0004
id3tech+1252	0.0111 ± 0.0002	0.0205 ± 0.0003	0.0369 ± 0.0004
innovatrics+0017	0.0048 ± 0.0002	0.0102 ± 0.0002	0.0214 ± 0.0003
morpho+0108	0.0047 ± 0.0002	0.0080 ± 0.0002	0.0130 ± 0.0003
morpho+0109	0.0040 ± 0.0001	0.0070 ± 0.0002	0.0118 ± 0.0002
nec+8210	0.0043 ± 0.0001	0.0063 ± 0.0002	0.0092 ± 0.0002
Neurotechnology+0204	0.0114 ± 0.0002	0.0206 ± 0.0003	0.0357 ± 0.0004
IDWorld+0005	0.0251 ± 0.0004	0.0462 ± 0.0005	0.0724 ± 0.0006

Table 5: *Left index finger FNMRs at various FMRs when IDWorld+0005 and MINEX III-compliant matchers compare templates created by template generator IDWorld+0005.*

Matcher	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
005B+0015	0.0142 ± 0.0003	0.0212 ± 0.0003	0.0318 ± 0.0004
aatec+0201	0.0142 ± 0.0003	0.0232 ± 0.0003	0.0364 ± 0.0004
aatec+0300	0.0157 ± 0.0003	0.0251 ± 0.0004	0.0385 ± 0.0004
id3tech+1250	0.0190 ± 0.0003	0.0312 ± 0.0004	0.0491 ± 0.0005
id3tech+1252	0.0165 ± 0.0003	0.0293 ± 0.0004	0.0492 ± 0.0005
innovatrics+0017	0.0095 ± 0.0002	0.0197 ± 0.0003	0.0389 ± 0.0004
morpho+0108	0.0092 ± 0.0002	0.0152 ± 0.0003	0.0227 ± 0.0003
morpho+0109	0.0080 ± 0.0002	0.0135 ± 0.0003	0.0212 ± 0.0003
nec+8210	0.0066 ± 0.0002	0.0106 ± 0.0002	0.0155 ± 0.0003
Neurotechnology+0204	0.0173 ± 0.0003	0.0301 ± 0.0004	0.0489 ± 0.0005
IDWorld+0005	0.0371 ± 0.0004	0.0657 ± 0.0006	0.1019 ± 0.0007

Table 6: Two finger FNMRs at various FMRs when IDWorld+0005 and MINEX III-compliant matchers compare templates created by template generator IDWorld+0005.

Matcher	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
005B+0015	0.00117 ± 0.00008	0.0020 ± 0.0001	0.0031 ± 0.0001
aatec+0201	0.00154 ± 0.00009	0.0028 ± 0.0001	0.0052 ± 0.0002
aatec+0300	0.00149 ± 0.00009	0.0028 ± 0.0001	0.0048 ± 0.0002
id3tech+1250	0.00188 ± 0.00010	0.0033 ± 0.0001	0.0056 ± 0.0002
id3tech+1252	0.00142 ± 0.00008	0.0029 ± 0.0001	0.0053 ± 0.0002
innovatrics+0017	0.00022 ± 0.00003	0.00082 ± 0.00006	0.0022 ± 0.0001
morpho+0108	0.00025 ± 0.00004	0.00061 ± 0.00006	0.00127 ± 0.00008
morpho+0109	0.00013 ± 0.00003	0.00046 ± 0.00005	0.00099 ± 0.00007
nec+8210	0.00033 ± 0.00004	0.00069 ± 0.00006	0.00108 ± 0.00007
Neurotechnology+0204	0.00156 ± 0.00009	0.0031 ± 0.0001	0.0053 ± 0.0002
IDWorld+0005	0.0031 ± 0.0001	0.0069 ± 0.0002	0.0128 ± 0.0003

5 References

- [1] Jonathan N. Bradley, Christopher M. Brislawn, and Thomas Hopper. FBI wavelet/scalar quantization standard for gray-scale fingerprint image compression. In *SPIE, Visual Information Processing II*, 1961. [4](#)
- [2] George Doddington, Walter Liggett, Alvin Martin, Mark Przybocki, and Douglas Reynolds. Sheep, goats, lambs and wolves a statistical analysis of speaker performance in the nist 1998 speaker recognition evaluation. In *INTERNATIONAL CONFERENCE ON SPOKEN LANGUAGE PROCESSING*, 1998. [4](#)
- [3] Patrick Grother Elham Tabassi, George W. Quinn. When to fuse two biometrics. In *IEEE Computer Society on Computer Vision and Pattern Recognition, Workshop on Multi-Biometrics*, 2006. [4](#)
- [4] Robert Fontana, Giovanni Pistone, and Maria Rogantin. Classification of two-level factorial fractions. *Journal of Statistical Planning and Inference*, 87:149–172, 2000. [4](#)
- [5] P. Grother, M. McCabe, C. Watson, M. Indovina, W. Salamon, P. Flanagan, E. Tabassi, E. Newton, and C. Wilson. Performance and Interoperability of the INCITS 378 Fingerprint Template. Technical report, NIST, 2006. [5](#)
- [6] Olaf Henniger and Dirk Scheuermann. Minutiae template conformance and interoperability issues. In Arslan Brömme, Christoph Busch, and Detlef Hühnlein, editors, *BIOSIG*, volume 108 of *LNI*, pages 25–32. GI, 2007. [12](#)
- [7] A. Martin, G. Doddington, T. Kamm, M. Ordowski, and M. Przybocki. The DET curve in assessment of detection task performance. In *Proc. Eurospeech*, pages 1895–1898, 1997. [4](#)
- [8] George W. Quinn. Evaluation of latent fingerprint technologies: Fusion. In *NIST Latent Fingerprint Testing Workshop Recognition, Workshop*, 2009. [4](#)
- [9] Elham Tabassi, Patrick Grother, Wayne Salamon, and Craig Watson. Minutiae interoperability. In Arslan Brömme, Christoph Busch, and Detlef Hühnlein, editors, *BIOSIG*, volume 155 of *LNI*, pages 13–30. GI, 2009. [12](#), [14](#)
- [10] Edwin B. Wilson. Probable Inference, the Law of Succession, and Statistical Inference. *Journal of the American Statistical Association*, 22(158):209–212, 1927. [4](#)